Stellar Evolution

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1 Introduction

Stars are like people - they have a life that comes in distinct stages. Stars are "born" from clouds of gas, spending their adolescent years going through many changes. Then, they spend the majority of their lives as an adult on the **main sequence** before they eventually meet their end. Furthermore, where and how a star is born determines where they will end up. Some go out with an explosive bang, while others slowly dim before they even had a chance to shine.

Together, these transitions are known collectively as **stellar evolution** (this differs from biological evolution in that it describes changes in a single individual rather than a population). Since a star goes through many stages in their life, and these stages differ depending on the **mass** and **metallicity** a star begins with, it can be daunting to try and understand their lives. This handout will begin by explaining some general properties of stars. With these tools in hand, it will be easier to understand and predict how a star evolves

- **Bolded** terms are important
- Gray text is advanced info and can be safely skipped without impacting understanding

2 Background

2.1 Hydrostatic Equilibrium

Stars are under the constant influence of two forces: **gravity** pulling inwards and **radiation pressure** pushing outwards. When these two forces are balanced, the star is said to be in **hydrostatic equilibrium**. Some general trends:

- **Radiation pressure increases with temperature**. Radiation pressure is an outward force caused by the momentum of photons produced in the stellar interior being transferred to matter within the star, causing the star to expand. Higher temperatures means more fusion occurs and more photons are produced, resulting in greater radiation pressure.
  - In addition, as temperature increases, molecules in the Sun vibrate more, increasing the gas pressure. Therefore, higher temperatures cause both radiation pressure and gas pressure to increase, creating an outward force that pushes the star to expand.
- **Getting smaller, getting hotter:** As a star contracts, gravitational potential energy is converted to kinetic energy, increasing the temperature (average kinetic energy) of the star.
  - This is a result of the **Virial theorem**, which states that for most systems in equilibrium, $E_k = -\frac{1}{2}E_p$. This is because the energy conditions in such a system are similar to those found in a circular orbit. This trick is useful for simplifying energy scenarios and can show up in competitions like USAAAO.
- **Getting larger, getting cooler:** This is the opposite of the previous idea. A star's surface will get cooler as it expands.

- **More massive stars evolve quicker:** This is because they will burn through their fuel supply quicker (There is a **mass-luminosity relation** $L \propto M^{3.5}$, so luminosity [or the rate at which fuel is used] increases faster than mass [or the amount of fuel] does) and move on to the next stage of their life. This also means **more massive stars have shorter life spans**!

- **Stefan-Boltzmann Law:** Describes the power radiated by an object; used to describe luminosity of a star.

$$P = \epsilon \sigma A T^4$$

  - P is the absolute luminosity,
  - ϵ is the emissivity, and is 1 for perfect blackbodies like stars (NOTE: while stars are often approximated as perfect blackbodies, they are not actually perfect blackbodies)
  - $\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$ is the Stefan-Boltzmann constant.
  - A is the surface area, and is $4\pi R^2$ for spheres
  - T^4 is the temperature raised to the 4th power.

  This means that

  - **Bigger stars appear brighter**.
  - **Hotter stars appear brighter**.

2.2 HR Diagram

In order to make sense of stars, we first must develop a system to classify them. The two main properties used to classify stars stars are absolute luminosity (since we can directly measure apparent magnitude and then calculate absolute magnitude, and thus luminosity, if we know the distance to the star), and temperature (which can be found from the star's emission spectra). There are a few important classification schemes you should know:

- **Harvard spectral classification:** Uses the strength of H I Balmer lines to classify stars based on temperature.

  Due to the Stefan-Boltzmann law, a higher temperature should result in stronger H I lines, since the star becomes more luminous. So, stars were assigned to classes A through Z where A had the strongest lines and believed to be hottest, while Z had the weakest and believed to be coldest. Over time, most of the classes fell out of use and only a few remain.

  However, **Annie Jump Cannon** realized while looking at spectra of stars that some stars that had weak HI lines also had really strong H II lines. This means that these stars are so hot that it *ionized* the H, which is why H II (singly ionized) lines were strong and H I (neutral) lines were weak. She reorganized the classes based on temperature and came up with a mnemonic to help remember the order:

  Oh **B**e **A** **F**ine **G**irl, **K**iss **M**e

The dwarf star classifications **L, T, Y** were later added to the end, so the order of stars from hottest to coolest is **OBAFGKMLTY**

A single digit is added after the letter to specify how hot it is within that class, with 0 being the hottest and 9 being the coolest.

Thus, a B0 star is hotter than a B3 star, but a B5 star is hotter than an A2 star.

Other weird stars:

- **W:** Wolf-Rayet stars, these are young, very hot and large stars that lack H
- **C:** Carbon stars, these stars have a lot of carbon in their atmosphere and often make up Mira variables
- **S:** These stars are intermediate between normal stars and C stars

- **Yerkes spectral classification:** Classifies based on size of star.
  - Ia: luminous supergiants
  - Ib: supergiants
  - II: luminous giants
  - III: giants
  - IV: subgiants
  - V: main sequence stars
  - VI: subdwarfs
  - VII: white dwarfs

- **Morgan-Keenen System:** The MK system combines the Harvard and Yerkes systems into one name.

  For example, our Sun is G2V, making it a G-type main sequence star.

- **Populations:** A star is formed from a cloud of interstellar gas - the remnants of a previous star that exploded. Since a star converts light elements like H to heavier elements during its lifetime via fusion, we would expect the stars of later generations to start off with all of the metals (In astronomy, metal refers to any element heavier than He) formed by its predecessors. With this idea, we can split stars up into generations, known as **populations**:
  - **Population I**: These are stars like our Sun that formed recently. Therefore they are young, typically bluer in color, and have a high metallicity (Note: a "high" metallicity is still only around 1.4%)
  - **Population II**: These are stars that formed a long time ago. Therefore, they are old, typically redder, and have a low metallicity.
  - **Population III**: These are a hypothetical class of stars that formed soon after the Big Bang. These stars have virtually no metal and are incredibly old and red. None have been observed yet.

Now we can talk about how we classify stars. Stars are plotted on the **Hertzsprung-Russell (HR) diagram** where the x-axis is temperature, with hottest on the left and coolest on the right, and the y-axis is absolute magnitude or luminosity, with brighter stars higher up and dimmer stars on the bottom.

- Instead of temperature, the x-axis may have spectral class, color, or B-V index (which all depend on temperature).
- It may seem weird that temperature is hottest on the left, but it is done this way so that the diagram looks nicer and the main sequence doesn't just abruptly end, as you will soon see.
- Stars of the same size follow a *diagonal* relationship. Due to the Stefan-Boltzmann Law, hotter stars appear brighter. So, for a given size, we would expect luminosity to increase (moving **up**) as temperature increases (moving **left**). This also means that as a star increases in size, we would expect it to move perpendicular to these diagonals, so **up** and the the **right**. This relationship becomes apparent when we see where red giants and white dwarfs are located and when we track the evolution of stars on the HR diagram.

If we look at all the stars in the sky and plot them, we notice that they fit into certain groups on the HR diagram:

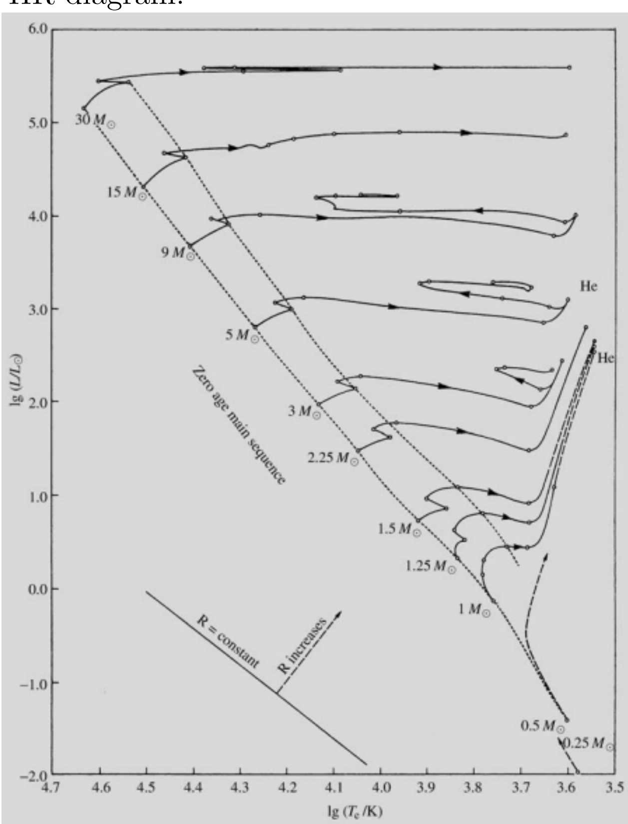


Figure 1: The HR diagram, showing the evolutionary path of stars of different masses. In the bottom left-corner, there are two lines showing the diagonal relationship a star's size follows on the HR diagram. (Source: Fundamental Astronomy by Karttunen)

- **Main sequence**: This is where stars will spend most of their life. It is a nearly straight line that runs from the upper-left (blue giant stars) to lower-right corners (red dwarf stars) of the diagram.

  When we look at the sky, 90% of stars are main-sequence, which means that a star will spend about 90% of its life on the main-sequence. Why is this true? It's like a census! If we take a snapshot of the population by recording everyone's age, we'll see that there are some adolescents and senior citizens, but a LOT of working age adults. This is because people spend most of their lives working. Similarly, if we take a snapshot of stars by looking at the sky, most of them are main-sequence because they spend most of their lives on the main-sequence.

  The lower end of the main-sequence is more densely populated than the top end because high mass stars evolve quicker and spend less time on the main sequence.

- **Red giants and supergiants**: These stars are clumped in the upper-right corner of the diagram. The red giants are really large, which is why they are so bright, and since as stars get larger, their surface gets cooler, these large stars have very cool, or red surfaces.
- **White dwarfs**: These stars are clumped in the lower-left corner of the diagram and are the opposite of red giants. They are small, and therefore not very luminous. However, since getting smaller gets hotter, these stars are also very hot, giving them their white color.
- **Instability strip**: This is where **variable stars** are located - stars that are unstable and vary regularly in brightness. More on this in the variable stars handout.

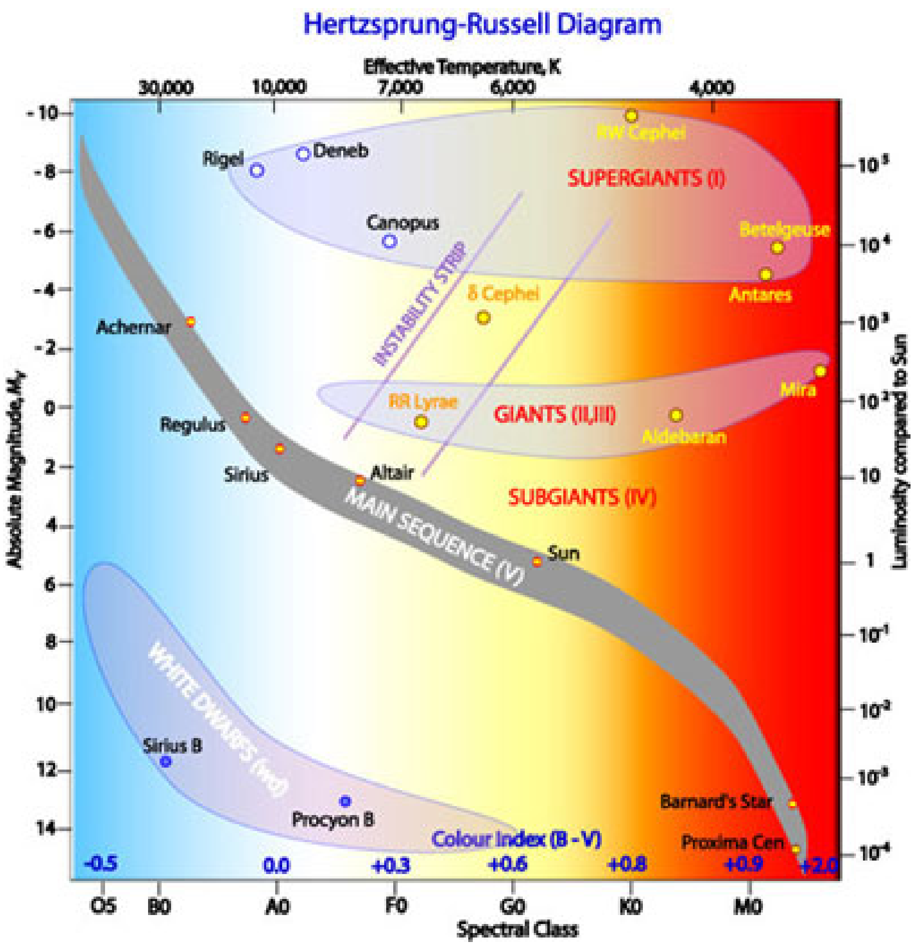


Figure 2: The HR diagram, showing where stars are located (Source: COSMOS)

2.3 Fusion

To see how stars survive, we first need to understand where they get their energy from. Simply put, **nuclear fusion** converts mass into energy by the relation $E = mc^2$. By smashing smaller nuclei together, stars can make heavier elements (**nucleosynthesis**). Generally, fusion of heavier elements occur at higher temperatures because more energy is required to smash them together (since there are more protons so there is a higher **Coloumb barrier**... **quantum tunneling** helps overcome this barrier). There are many different types of fusion that are classified based on what elements are involved, but only the main 3 are important:

- **Deuterium fusion**

  This process occurs in protostars, slowing their contraction by increasing radiation pressure. These protostars are not considered stars yet because they're not fusing H.

  Occurs at lower temperature than proton-proton chain. Acts mainly as a thermostat, regulating the contraction of the protostar, rather than a source of energy

  Possible in planets $> 13M_J$

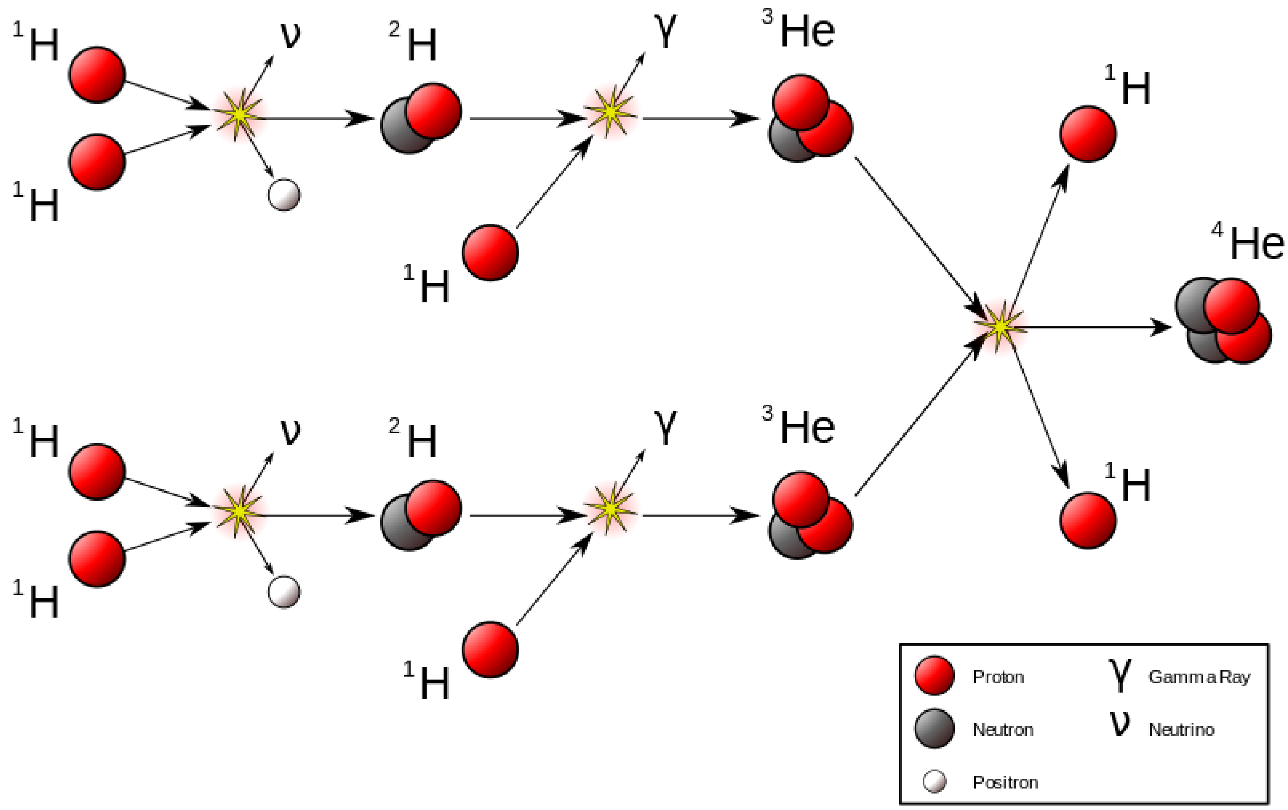


Figure 3: Proton-proton chain (pp1) (Source: Wikipedia)

- **Proton-proton chain:** 4H → He

  As the name says, we take H nuclei (protons) and smash them together one after another (a chain of reactions) to eventually make He.

  The first step releases **neutrinos** ν_e (which escape the star, carrying away energy) and a **positron** e^+, which quickly combines with an electron to release 2 **gamma rays** γ

  The second step releases a gamma ray.

  Dominant process in Sun-like main sequence stars.

  There are actually many reactions that could happen (pp1, pp2, pp3, pp4[Hep], PEP) but usually pp1 is used.

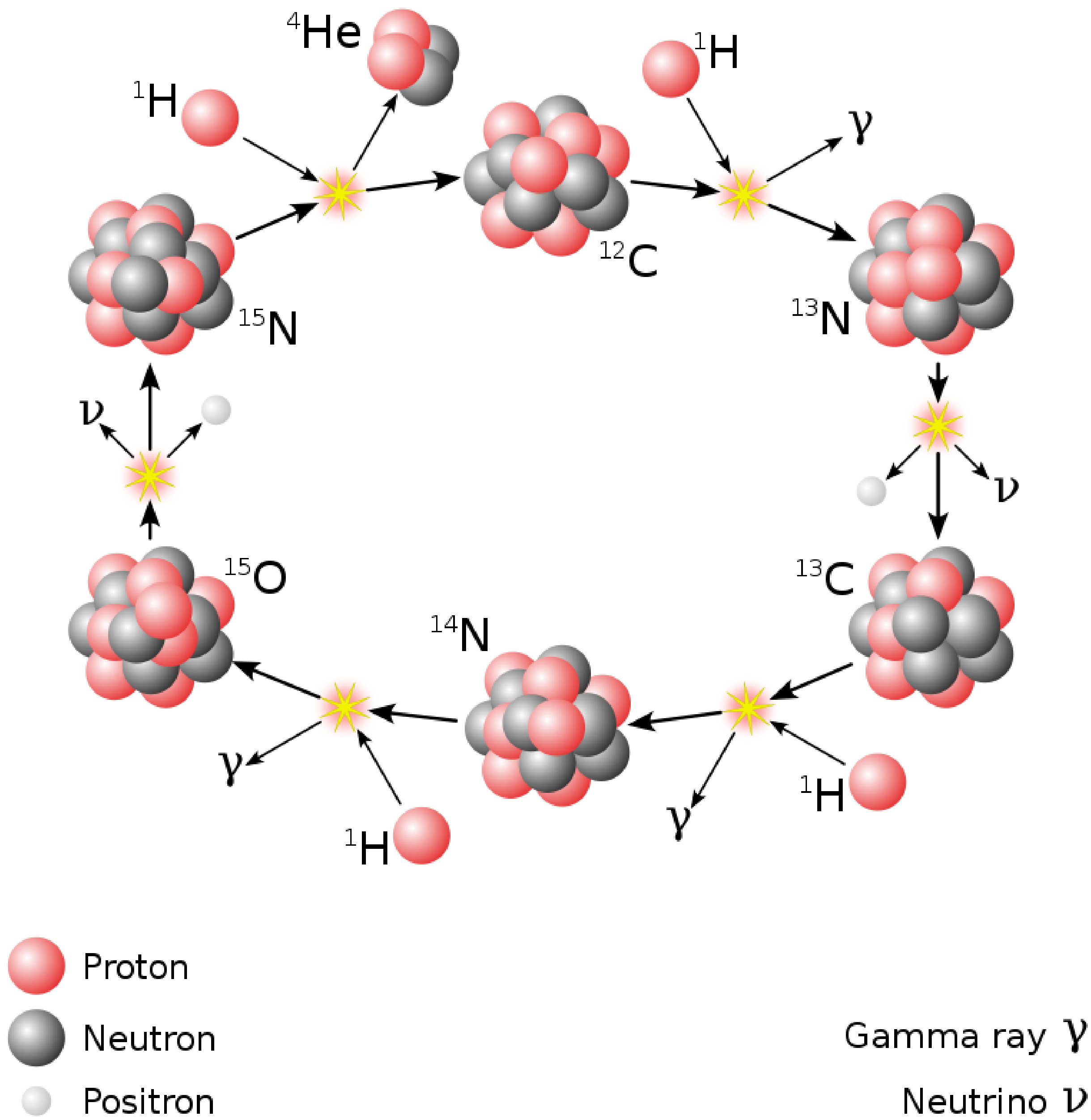


Figure 4: CNO cycle (Source: Wikipedia)

- **CNO Cycle:** 4H → He using a catalyst!

  $T > 20$ mil K

  This is an alternative to the proton-proton chain that uses a catalyst, C N and O.

  Dominant process in main sequence stars $> 1.3 M_\odot$ (some sources say $1.5 M_\odot$) because CNO cycle reaction increases more rapidly with temperature than proton-proton chain (17th power vs 4th).

  AKA: Bethe-Weizsäcker cycle

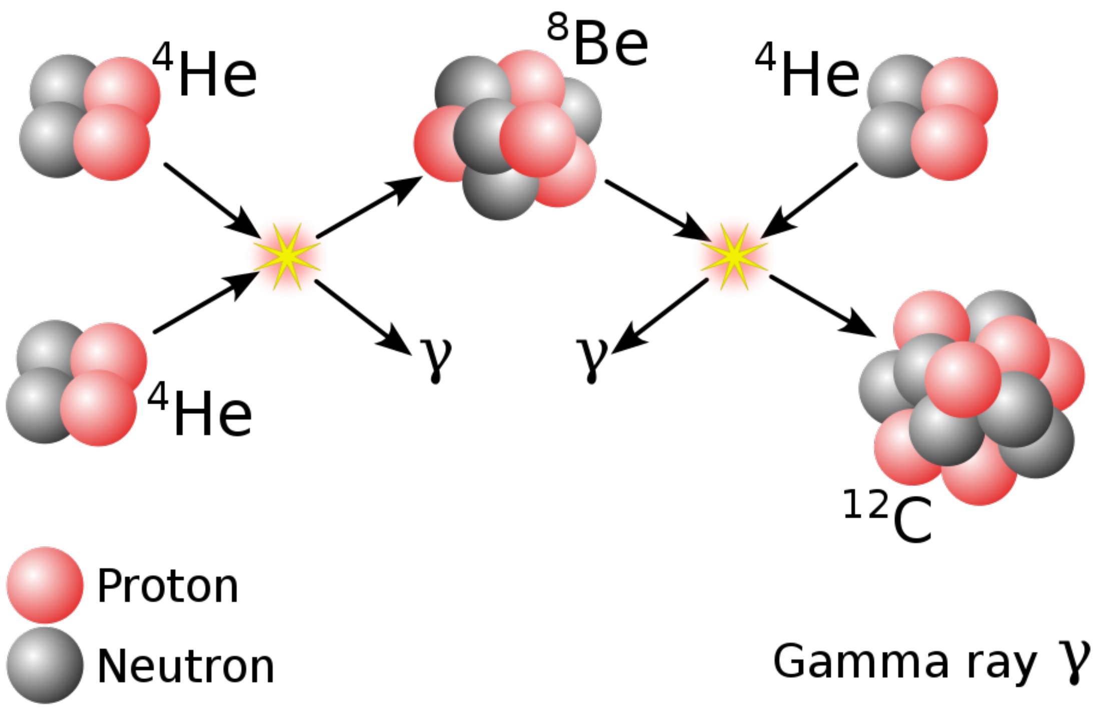


Figure 5: Triple alpha process (Source: Wikipedia)

- **Triple alpha process:** 3He → C

  $T > 10^8$ K

  A **Helium flash** occurs when stars begin this process, marking the transition from the red giant branch to horizontal branch (explained later)

Any process that has "burning" in the name just means the star keeps on fusing that element to make heavier elements until it runs out.

- **Alpha process (Helium burning):** Keep adding He to make heavier elements
- **Carbon burning:** After He is used up, keep adding C to make heavier elements

  $T > (5-8) \times 10^{10}$ K

  Occurs in stars $> 8M_\odot$
- **Neon burning:** Same thing after C, keep adding Ne
- **Oxygen burning:** Even though O is lighter than Ne, comes after because O_{16} is **doubly magic**
- **Silicon burning:** Makes ^{56}Ni and ^{56}Fe

  The production of elements heavier than Fe requires an input of energy, and thus does not occur except during explosive stellar deaths.
- **Lithium burning:** Occurs in brown dwarfs, which are cool enough to still have Li

The next few processes are used to make elements heavier than Fe and follow the **nuclear/proton drip line** (when atoms pass this limit they begin to decay).

- **r-process:** Rapidly adds neutrons 1 after another to make elements heavier than Fe
  Occurs when stars explode.
- **s-process:** Adds neutrons slowly enough that β^- decay can happen
  Occurs in AGB stars (explained later)
- **rp-process:** Rapidly adds protons
  Tin-antimony-tellurium cycle is the upper limit.
- **p-process:** Creates neutron-deficient isotopes, process currently unkown

At $T > 10^9 K$, photons contain enough energy that they can break up nuclei into lighter elements (**photodisintegration**)

2.4 Stellar Interior

Stars are like onions, they have layers.

There are 3 main layers to a star, as you go deeper, the layers get hotter:

- **Core**: Where most of the fusion happens.
- Zones where energy is transferred to the outside.
  - **Conduction**: only happens in very dense objects, like white dwarfs and neutron stars (More later)
  - **Convection** takes over **radiation** when the temperature gradient becomes too high or the star is too opaque for radiation to occur. Convection also mixes the contents within the star, causing higher abundances of heavy metals in outer layers.
  - $.08M_\odot < M < .26M_\odot$: These stars are convective throughout.
  - **Low-mass (Sun-like) stars**: They have a **radiative core** since the pp chain occurs throughout the core and isn't super concentrated. This creates a low temperature gradient, allowing for radiation.

    They have a **convective envelope** since the low temperature makes the envelope opaque, preventing radiation from transfering the energy.
  - **High-mass stars** ($> 1.5M_\odot$): They have a **convective core** because the CNO cycle is temperature-sensitive and concentrated in the center where it is hottest. This makes a steep temperature gradient, requiring convection in the inner layers.

    They have a **radiative envelope** because no nuclear reactions are occuring in the envelope and it is hot enough to allow for radiation.

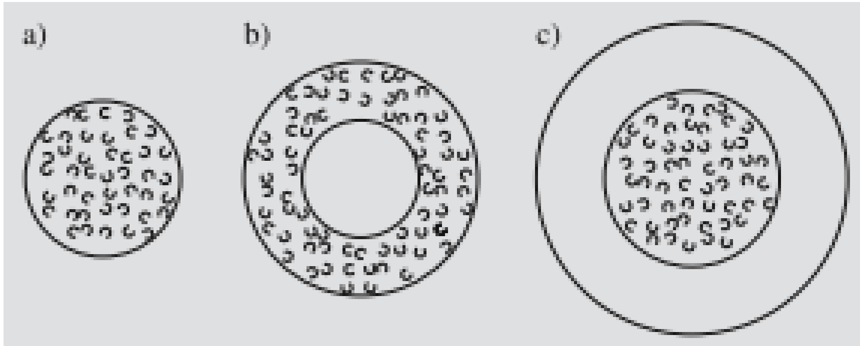


Figure 6: Layers in stars of different masses. C's represent convective layers while unfilled layers are radiative (Source: Fundamental Astronomy by Karttunen)

- The atmosphere. The layers of the atmosphere from innermost to outermost are:
  - **Photosphere**: This is the part of a star we actually see. When we talk about temperature of star, we are talking about the surface temperature of the photosphere.
  - **Chromosphere**: This is an outer, reddish layer that is only seen during solar eclipses.
  - **Corona**: This layer extends very far but has very few particles. Since temperature is the average kinetic energy of particles, the corona is the hottest layer since it has so few particles to carry the energy.

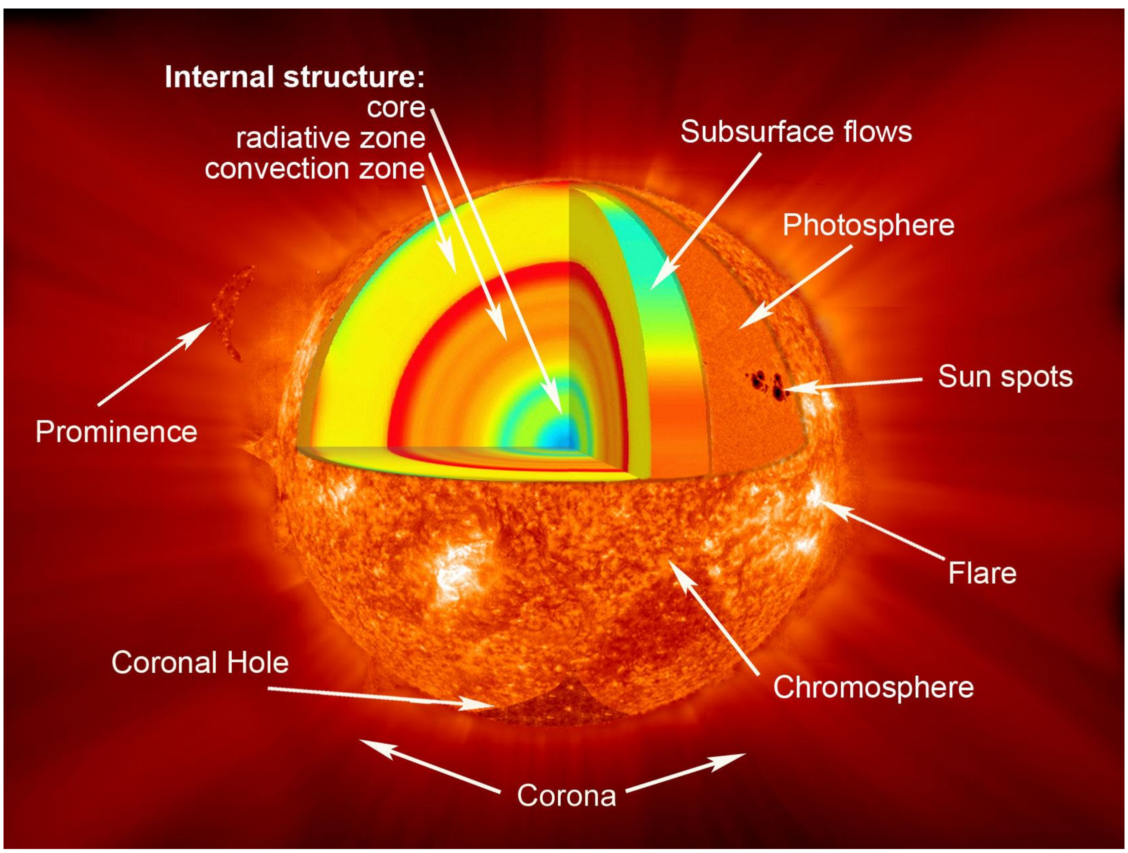


Figure 7: Layers of the Sun and surface features (Source: NASA)

Now that we understand how stars work and can describe them using the HR diagram, we have all the tools necessary to track the evolution of a star through its life.

3 Stellar Evolution

Through this section you will see that a star will stay in its current phase until a change happens. Whether it is a change in composition or energy transfer, it will cause the star to advance to the next stage of its life. For each section, we will first describe what happens for a Sun-like star and then see how the process changes when mass or metallicity changes.

3.1 Protostars: A Star is Born

A star isn't officially a star until it starts fusing hydrogen in its core. Before that, it is known as a **protostar**. Protostars form when a cloud of interstellar gas (known as a **stellar nebula**) contracts. As it gets smaller, it gets hotter, causing the temperature to rise until the cloud is fully convective and becomes a protostar. At this point, we start plotting it on the HR diagram. Since fusion has not yet begun, the surface temperature is cool. Also, the gas cloud is large, so it has a high luminosity. Thus, our protostar is in the upper-right corner of the HR diagram.

At this point, the star begins to follow a line in the HR diagram called the **Hayashi track** going basically straight down as the protostar continues to contract and get hotter. Eventually, the center gets hot enough that its opacity decreases and radiation can begin in the core. At this point, the protostar gets off the Hayashi track and begins going left on the HR diagram along the **Henyey track** as its surface temperature increases. It continues along this path until hydrogen fusion begins in the core and the protostar becomes a star on the main-sequence.

For higher mass stars, the change from Hayashi to Henyey happens earlier, so their protostar will appear to follow a mostly horizontal path while lower mass protostars will apear to follow a mostly vertical path.

A **T Tauri star** is an example of a protostar in this phase. It has a high abundance of **Li**, indicating that they are new stars that have not reached temperatures high enough to destroy Li.

When these stars form, they develop an accretion disk around them. As a result, radiation is forced to exit on opposite sides that the disk does not obscure, creating polar jets of stellar wind. These jets may collide with clouds of gas and dust near a T Tauri star, creating small bright nebulae called **Herbig-Haro (HH) objects**.

Note: There is an interesting balance involved whenever we have matter accreting onto a stellar object. The more mass that accretes, the more energy the central object emits (or more friction in the accretion disk radiates more energy). The outflow of energy and stellar wind prevents more matter from accreting, starving the newborn star of fuel. Thus, protostars must reach a balance between accreting enough matter to produce enough energy to become a star, but not producing so much energy that it will drive away its food source. This also applies to other stellar objects that form accretion disks, such as black holes.

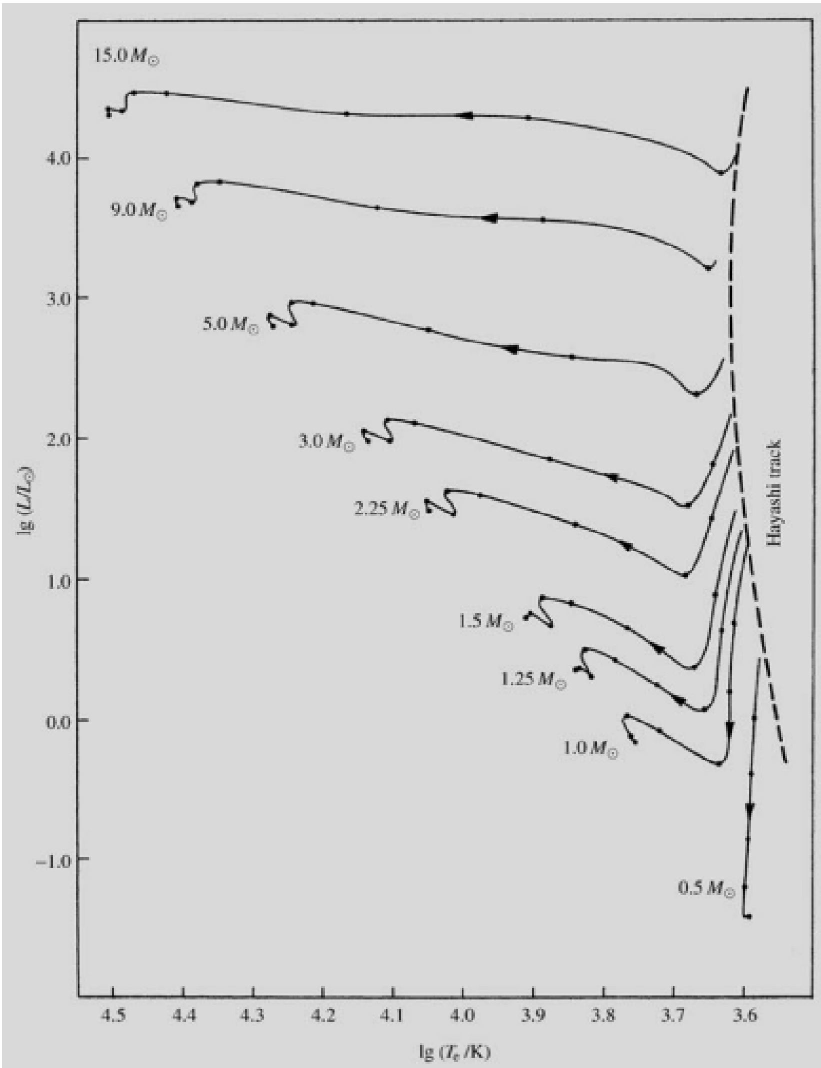


Figure 8 (left): the path of a protostar on the HR diagram.
(Source: Fundamental Astronomy by Karttunen)
Figure 9 (right): HH47, a Herbig-Haro object from a T Tauri star. The dark cloud in the middle is the accretion disk, obscuring the protostar from sight. The bright clouds on either end show the polar jets creating HH objects. (Source: Wikipedia)

3.2 Main Sequence

Stars spend most of their life on the main sequence, fusing **hydrogen** in their **core**. Low-mass stars (Sun-like) do so via the proton-proton chain, while high-mass stars do so via the CNO cycle, causing differences in their radiative and convective layers (described in 2.4 Stellar Interior).

3.3 Red Giant Branch (RGB)

As stars fuse hydrogen in their core, helium begins to accumulate in the center. Eventually, enough helium accumulates that the hydrogen core is replaced by a helium core! However, the core is still not hot enough for helium fusion to occur, so there is **hydrogen** fusion in a **shell** around the core.

Since a shell has more surface area than the core, fusion occurs at a faster rate than before, causing more radiation pressure to push out on the outer layers, making the star expand. The star grows rapidly in size, causing its luminosity to increase. As a result, the **surface** (the core continues to get hotter!) will *decrease* in temperature, causing the star to get redder. This is what makes a **red giant**.

On the HR diagram, stars on the red giant branch move up and to the right.

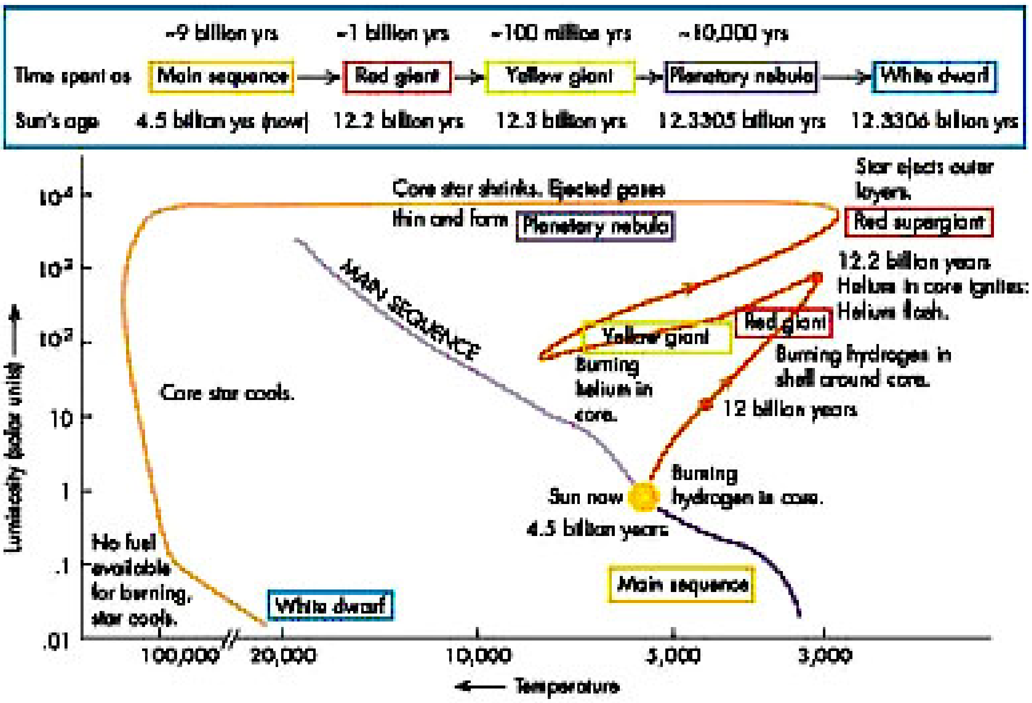


Figure 10: The evolutionary track of the Sun on the HR diagram. Here we can see that from the main sequence, the Sun travels along a slight curve known as the Subgiant branch before moving up and to the right along the Red Giant Branch. (Source: Chandra)

In low-mass (Sun-like) stars, the transition from main sequence to RGB is gradual and known as the **Subgiant branch**. Since high-mass stars evolve quickly, there is no noticable subgiant branch.

3.4 Horizontal Branch

As helium continues to accumulate in the core, eventually it gets hot enough for the **triple alpha process** to occur! This rapid start to **helium** fusion in the **core** causes an explosion in the core known as the **helium flash**. However, the energy from this explosion does not cause the star to expand. Instead, it is used to make the degenerate (basically just super dense) helium core expand and become not degenerate.

As a result, the star's size does not change much. In fact, luminosity may actually *decrease* as energy is used to expand the degenerate core. Surface temperature increases as this increase in fusion releases more energy. Therefore, on the HR diagram, our red giant will move to the left (and a bit down) along the **Horizontal branch**.

The helium flash is super important because it marks the transition from the RGB to the Horizontal branch! The horizontal branch is actually pretty complicated because it can take a bunch of different paths based on the star's mass and metallicity:

- **Low-mass (Sun-like)**: The horizontal branch is actually split into two segments, **red** and **blue**, which are separated by an **RR Lyrae gap** where the stars lie on the instability strip.

  In globular clusters with low metallicity, the blue horizontal branch is prominent.

  In solar metallicity stars, the horizontal branch is reduced to a short stump called the **red clump**.

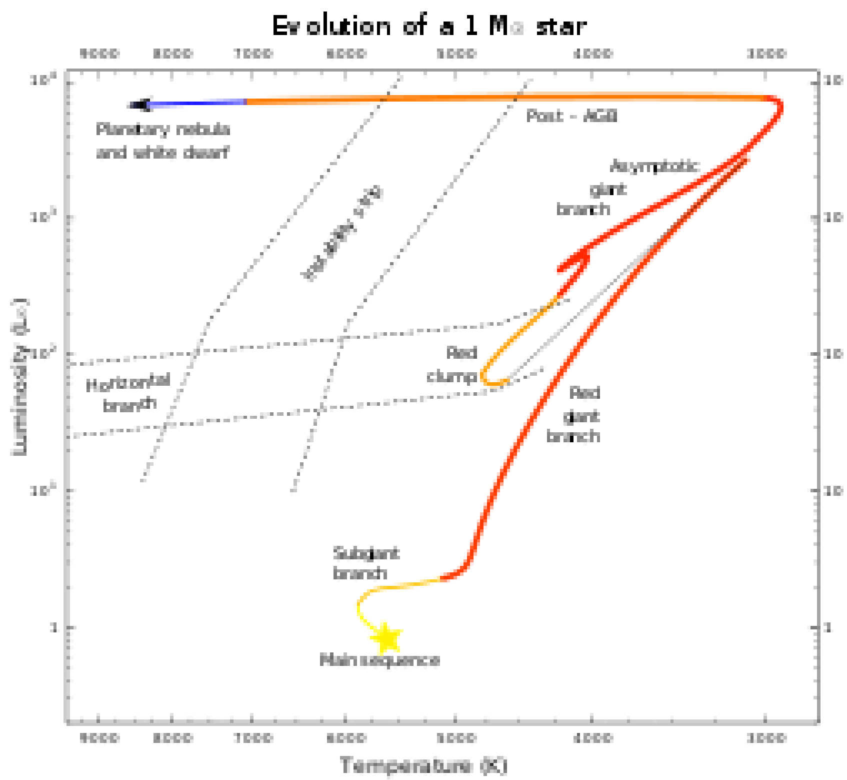


Figure 11: The evolutionary track of a Sun-like star on the HR diagram, showing the red clump.

- **Intermediate-mass** ($2.3M_\odot \leq M \leq 8M_\odot$): In these stars, the central temperature is high enough that the helium core is never degenerate. Instead, these stars will follow a **blue loop**, increasing in temperature towards bluer colors and then cooling back towards the RGB.

  The blue loop causes these stars to cross the instability strip where they become **Cepheid variables**.

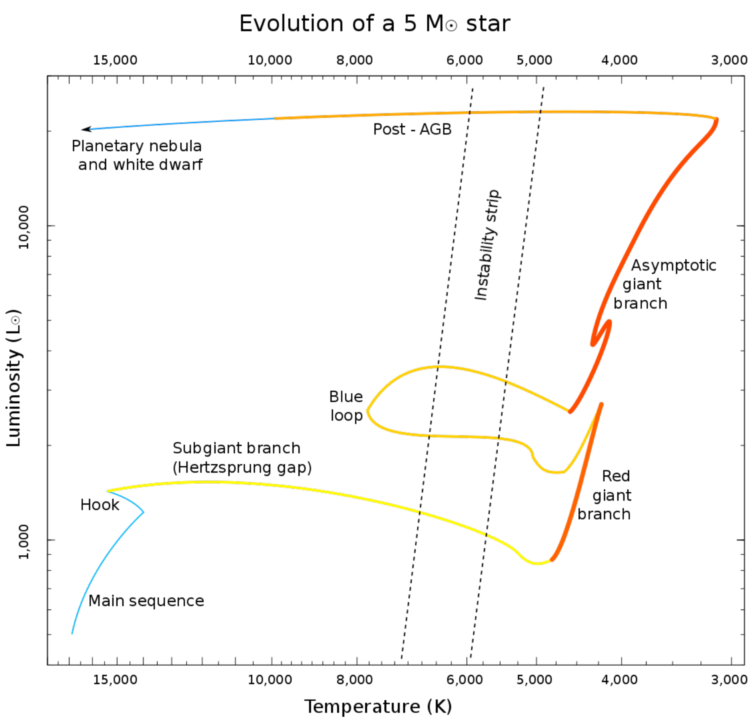


Figure 12: The evolutionary track of an intermediate-masss star on the HR diagram, showing the blue loop.

- **Massive stars**: These stars begin helium burning before they even reach the RGB. This causes the star to continue moving to the right to become a red supergiant. Stars in this phase have massive stellar winds and are known as **Luminous Blue Variables (LBVs)**. The LBVs may loose too much mass to become a red supergiant and will instead turn to the right to become a **Wolf-Rayet star**.

3.5 Asymptotic Giant Branch (AGB)

This stage is very similar to the RGB except instead of H, it involves He. Eventually, carbon from the triple alpha process will accumulate in the center and the helium core will be replaced by a carbon core! However, the core is still not hot enough for carbon burning to occur, so there is **helium** fusion in a **shell** around the core.

Since a shell has more surface area than the core, fusion occurs at a faster rate than before, causing the star to expand. This expansion causes surface temperatures to cool and luminosity to increase. This evolutionary process closely follows the RGB, and is therefore called the **asymptotic giant branch**.

After this stage, a lack of material to fuse means that the star will be unable to sustain itself and will die.

4 Dead Stars

4.1 Planetary Nebulae

Planetary Nebulae have nothing at all to do with planets! Early astronomers thought that these clouds of gas looked similar to planets, which is where their name comes from. When a star reaches the AGB, it begins to rely on helium shell fusion, a process that is unstable. In low-mass stars, the gravity is not strong enough to hold onto outer layers. As a result, the increased stellar wind from this instability is enough for the star to gently shed its outer layers, leaving behind a white dwarf core. UV radiation from the core excites the ejected layers of gas, creating the pretty colors we associate with planetary nebulae.

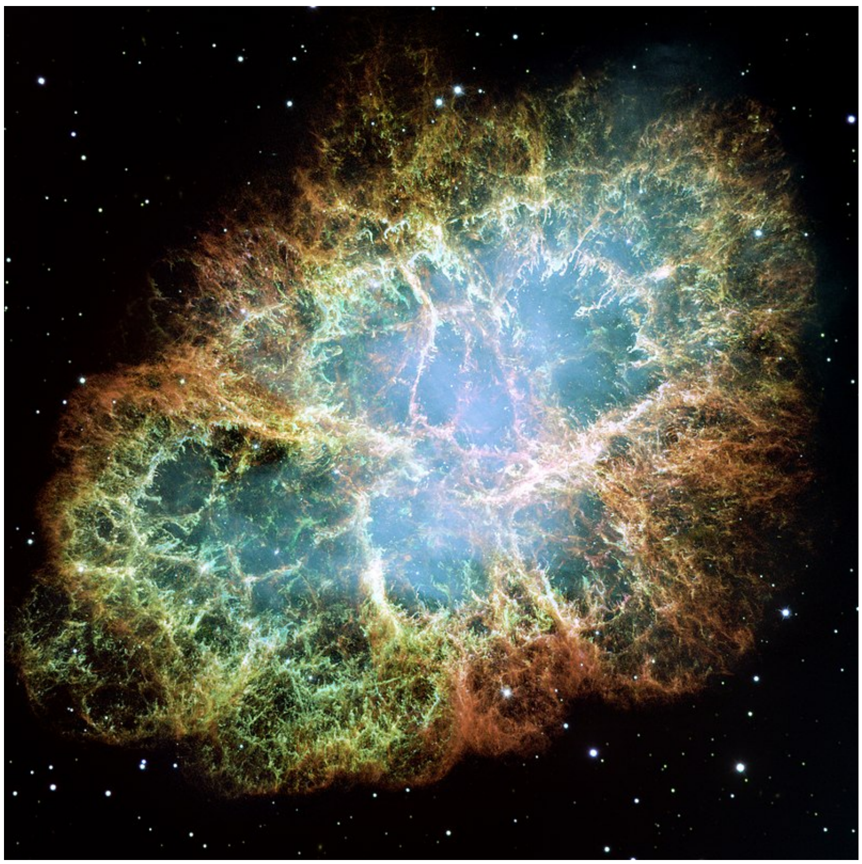


Figure 13: Crab Nebula, the remnant of one of the earliest recorded supernovae, SN 1054. (Source: Wikipedia)

4.2 Supernovae

In more massive stars, as they reach the end of the AGB and can no longer perform fusion, gravity takes over radiation pressure and the star begins to contract. The star continues to contract until it hits a degenerate core that can not contract any further. When the outer layers of the star hit this “wall”, they bounce off and go exploding into space! This explosion of a star near the end of its life is known as a **nova**. When a supermassive star dies, the explosion is super bright and known as a **supernovae**. There is an even brighter type of explosion called a **kilonova**, but this occurs during a binary neutron star or black hole merger.

Supernovae are classified based on what elements are found in their spectra:

- **Type I:** no H
  - **Type Ia:** no H, no He, strong Si II
    Caused by a **white dwarf** exceeding the **Chandrasekhar limit**.

    Type Ia supernovae are used as "**standard candles**" because they all have the same peak brightness **(-19.5)**. The reason for this consistency is that a white dwarf will always have the same mass ($1.44M_\odot$) when it collapses. This makes them insanely useful for determining distances to far-away places.

    Light comes from ^{56}**Ni** decaying to ^{56}**Co** to ^{56}**Fe**
  - **Type Ib:** no H, strong He, no Si
  - **Type Ic:** no H, no He, no Si
    Both Type Ib and Type Ic supernovae are likely caused by a massive **WR star** collapsing.
- **Type II:** has H

  Caused by a supermassive star collapsing.
  - **Type II-P:** has a **plateau** in its light curve
  - **Type II-L:** loses H faster than a Type II-P supernova, so has a linear decline instead of a plateau.
  - **Type IIn:** has narrow emission lines because the surrounding cloud is dense.
  - **Type IIb:** starts off as a Type II supernovae but transitions into a type Ib supernovae as H is lost.

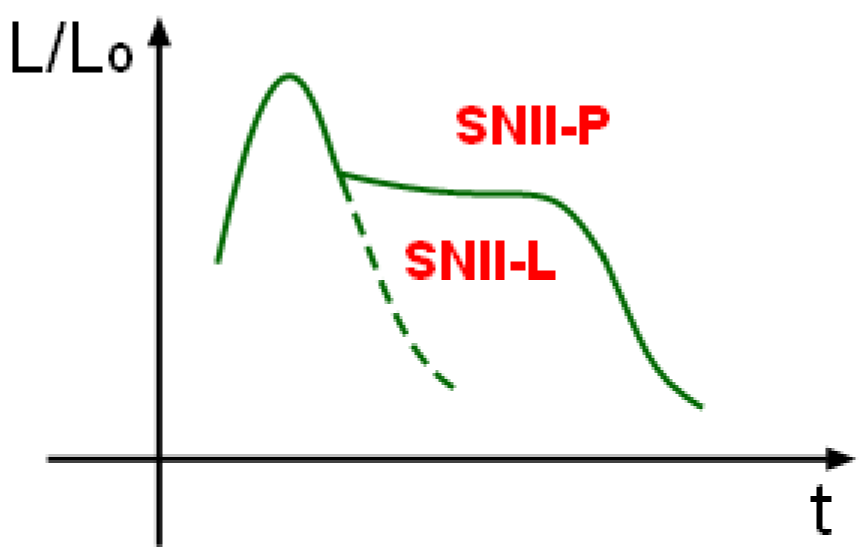


Figure 14: Light curve of Type II SN. (Source: Wikipedia)

4.3 White Dwarfs (WD)

At the end of the AGB, radiation pressure is no longer enough to counter gravity and the star contracts. It continues to contract until the atoms within the core get so close to each other that the electrons begin to overlap. Due to the **Pauli exclusion principle**, it is not possible for two electrons to occupy the same space. Thus, there is an **electron degeneracy pressure** preventing further contraction. A **white dwarf** is sustained by electron degeneracy pressure.

In the explosion that forms a white dwarf, the outer layers bounce off the degenerate core, so only a **C-O** core remains. In higher mass stars, C fusion occurs and the white dwarf is a **O-Ne white dwarf**.

A white dwarf can exist on its own and slowly radiate away energy until its surface cools (moving to the right on the HR diagram) to the point that the white dwarf stops emitting light and becomes a **black dwarf**. However, this process takes so long that it is believed no black dwarfs exist yet.

Alternatively, many white dwarfs are found in binary systems. Here, the white dwarf will accrete mass from its companion star, creating short bursts of energy known as **recurrent novae**. If the white dwarf accretes enough mass that it exceeds the **Chandrasekhar limit** ($1.44M_\odot$), then electron degeneracy pressure no longer becomes enough to resist the force of gravity. The white dwarf will continue to contract, pushing the atoms so close together that electrons will begin to combine into protons to form neutrons. This process releases a large amount of energy, creating a **Type Ia supernovae** and leaving behind a core of neutrons.

4.4 Neutron stars

If a star is massive enough ($> 8M_\odot$ at the end), then electron degeneracy pressure will not be enough to counter the force of gravity. At this point, only a core of neutrons will remain after the supernovae - a **neutron star**. Although most sources will say that **neutron degeneracy pressure**, a concept similar to electron degeneracy pressure but for neutrons, keeps a neutron star from further collapse, this is not true! For $M > .7M_\odot$, repulsive nuclear forces prevent further collapse rather than neutron degeneracy pressure.

Generally, the more massive an object is, the more intense radiation it will produce. **Neutron stars and black holes are the main source of X-rays we detect in space!** This is usually caused by intense friction in the **accretion disk** (matter falling into the central body forms a disk around it) raising the matter to super high temperatures, resulting in X-rays.

Neutron stars also have a mass limit of their own, known as the **Tolman-Oppenheimer-Volkoff (TOV) limit** and is $\sim 2.5M\odot$ most sources say $3M_\odot$ is a safe upper limit. If the neutron star exceeds this mass, nuclear forces can no longer counter gravity and the neutron star will collapse into a **black hole**.

When a star collapses into a neutron star, the **conservation of angular momentum** says that the star must spin faster like how a figure skater spins quicker when they pull their arms in. As a result, some neutron stars spin *really* fast.

In addition, magnetic fields must be conserved! So, as a neutron star collapses, the magnetic fields get stronger. This magnetic field prevents ejected material from escaping from the sides and forces it to exit through the poles in **astrophysical jets**. In addition, since neutron stars spin, where these jets point changes over time. If one of the jets happens to hit Earth, we observe it as a bright flash of light. As the neutron star spins around, we see these flashes happen at regular intervals. In this case, the neutron star is called a **pulsar**. If a pulsar spins *really* quickly, it is known as a **millisecond pulsar**. A pulsar with an incredibly strong magnetic field is called a **magnetar**.

Since the neutron star is radiating away energy, that causes its rotation to slow down over time (**spin-down**). However, the neutron star can increase its rotation speed by accreting mass (**spin-up**). A sudden spin-up is known as a **glitch** and its cause is unknown. Likewise, a sudden spin-down is an **anti-glitch**.

Although the contents of the interior of a neutron star is unkown, it is believed that the crust is made up of a substance called **nuclear pasta**, which comes in **gnocchi, spaghetti, lasagna, bucatini** and **Swiss cheese** phases. If it exists, nuclear pasta would be the strongest material in the universe.

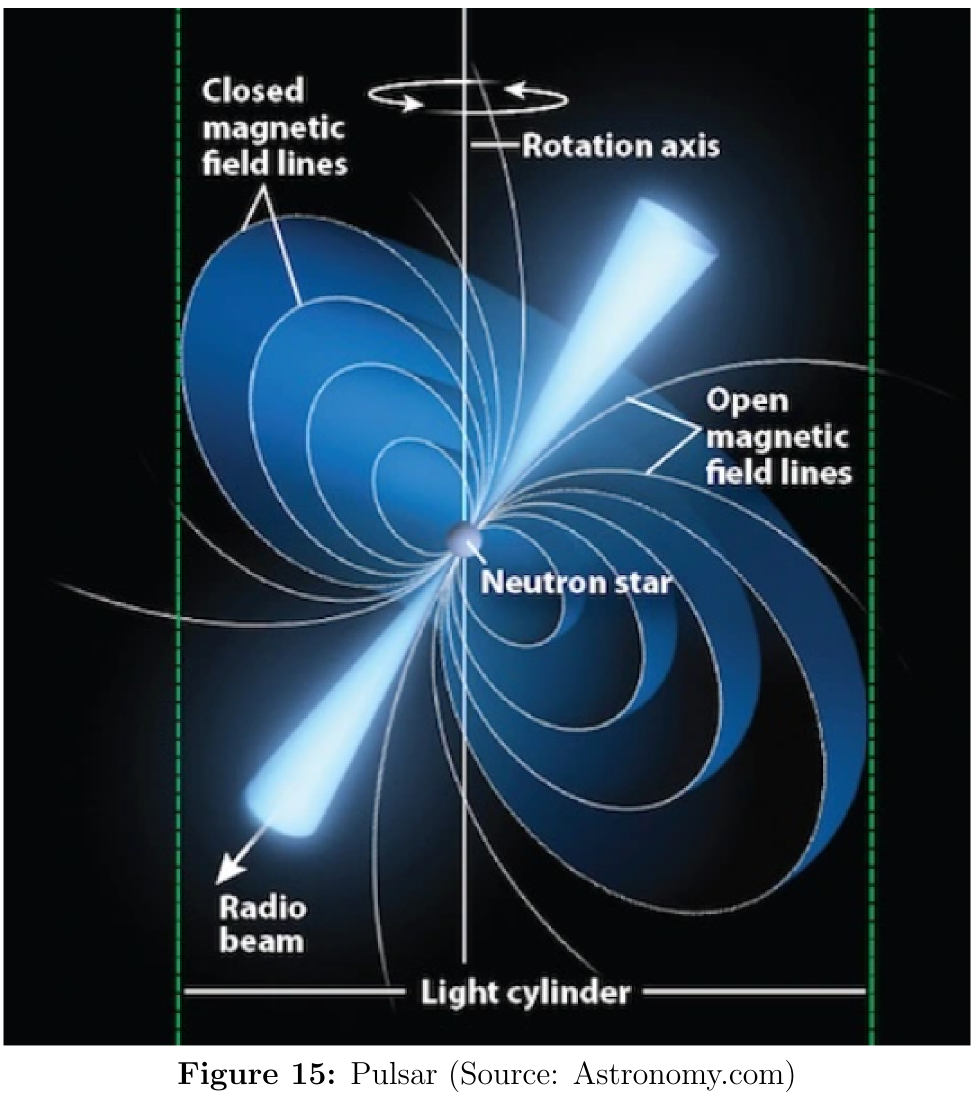


Figure 15: Pulsar (Source: Astronomy.com)

4.5 Black holes

If a star is massive enough, then no force will be able to resist the collapse caused by gravity. The star will continue to collapse, getting smaller and smaller, until all that mass is squished into a single point in space known as a **singularity**.

Black holes are so dense that nothing, not even light, can escape its gravitational pull. The **event horizon** marks the boundary at which light can no longer escape and is 1 **Schwarzschild radius** away from the center. We can calculate the value for the Schwarzschild radius by setting the escape velocity equal to the speed of light.

$$v_{escape} = \sqrt{\frac{2GM}{R}} = c, R = \frac{2GM}{c^2}$$

The **photon sphere**, or the minimum radius at which photons can have circular orbits around the black hole, is 1.5R. The **Innermost Stable Circular Orbit (ISCO)** for other particles is 3R.

The massive gravitational force of black holes allows for several interesting effects.

- As an object falls into a black hole, they will experience intense tidal forces. The side closer to the black hole will be pulled much more than the side away from the black hole, causing the object to elongate indefinitely, a process known as **spaghetification**.
- In addition, gravity near a black hole is so strong that it *bends* light around it, a process known as **gravitational lensing**. This allows observers to see objects located behind the black hole. Thus, it is impossible to actually *see* a black hole, since it envelops itself in space and light. However, we do have ways of detecting when a black hole is there. Gravitational lensing causes objects near the black hole to appear to multiply and spread out.
- In addition, due to general relativity, a black hole will bend the space around it. For reasons too complicated to discuss, it effectively causes light to travel a *longer* path in the same amount of space, causing its wavelength to appear to get *longer*, a process known as **gravitational redshift**.

Thus, when weird stuff happens, we know a black hole is there!

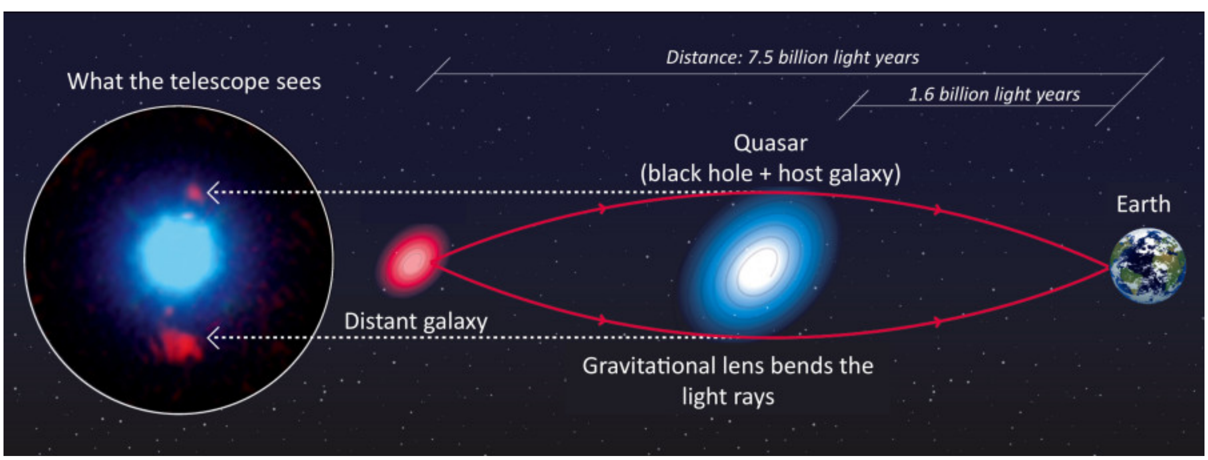


Figure 16: Gravitational lensing (Source: FastForward)

Since light can not escape a black hole, any information an object contains is lost when it goes into a black hole (since we have no way of observing that info). This idea that a black hole has no information is known as the **no hair theorem** (info is hair ig?). What happens to this information? I have no clue!

- The **Holographic principle** may offer an explanation. It is the theory that volume itself is an illusion. Information is contained to a 2D surface and the 3D world we experience is merely a "holographic projection" of that information, turning it into a form we can understand. In this sense, it may be that information that goes into a black hole is not lost. Instead, it is trapped on the 2D surface of the black hole. As a result, the information is not in a format we can process and so we perceive that information as being destroyed.

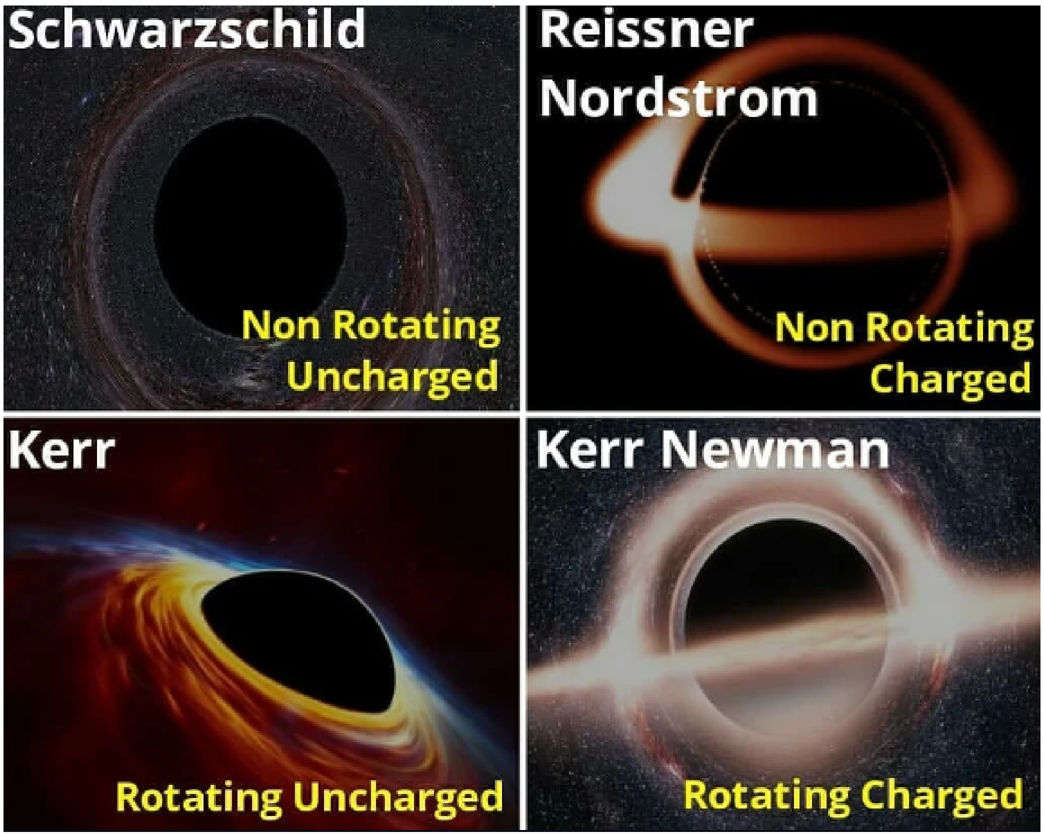


Figure 17: Black Hole types (Source: iFunny - messylustofscience)

The only 3 properties of a black hole that we can observe are **mass, spin,** and **charge**. We use these three properties to classify black holes:

- **Schwarzschild**: Defined only by mass.
- **Kerr**: Defined by mass and spin.

  Due to its spin, a Kerr black hole has a lot of weird properties like a 2^{nd} event horizon and an **ergosphere** - a region where space-time is literally pulled by a black hole.

  - In other words, NOTHING can remain *still* inside the ergosphere. This phenomenon is known as **frame-dragging** (AKA: Lense-Thirring effect).

  - Since frame-dragging causes space-time to move, if we had a Kerr black hole, we could theoretically shoot 2 photons into the ergosphere and have them split so one falls into the black hole while the other escapes with more momentum (gained from frame-dragging) than the 2 photons originally had. In other words, we can FARM ENERGY from a black hole! This is the **Penrose process**
  - The **Blandford-Znajek process** explains how **astrophysical jets** can form from black holes, leading to a lot of cool things like **quasars**!
  - Kerr black holes have also been involved in a lot of legit time travel theories which are waaay too complicated to discuss here. If you're curious, just watch Steins;Gate!!!

- **Reissner-Nordstrom**: Defined by mass and charge
- **Kerr-Newman**: Defined by mass, charge, and spin

Although light can not escape a black hole, they still emit radiation! This is called **Hawking radiation** and causes black holes to "evaporate", or get smaller. Since black holes radiate, we can assign a *temperature* to them. What's even weirder is that smaller black holes emit *more* Hawking radiation, so as a black hole evaporates, it gets hotter and evaporates even quicker until it eventually reaches an infinite temperature and has evaporated out of existence.

Why does Hawking radiation exist? It is likely due to **virtual particles**. The simple explanation is that at the event horizon, one virtual particle gets sucked in and one real particle gets expelled. We observe that real particle as Hawking radiation.

4.6 Binary systems

Binary systems of black holes or neutron stars are really weird phenomena, but their most interesting property is that as their orbits get closer together, they emit something known as **Gravitational waves**, allowing us to detect them!

Gravitational waves are wave-like distortions in space-time, effectively creating virtual "gravity wells" in space time and deforming things. However, these distortions are so subtle that in order to detect them, scientists use 4km long lasers at detectors like LIGO.

5 Binary stars

Everything we've dealt with up until now has involved solitary stars. However, binary systems (two stars orbiting around each other) complicate things. As we have seen, mass plays an important role in the evolution of a star, but as we will see, stars in a binary system can exchange mass!

Older sources say that as much as 85% of stars exist in a binary system, but with improving technology allowing us to observe more faint solitary stars, that estimate is likely to decrease. Still, binaries contain plenty of unique properties that make them important to study.

5.1 Types of binaries

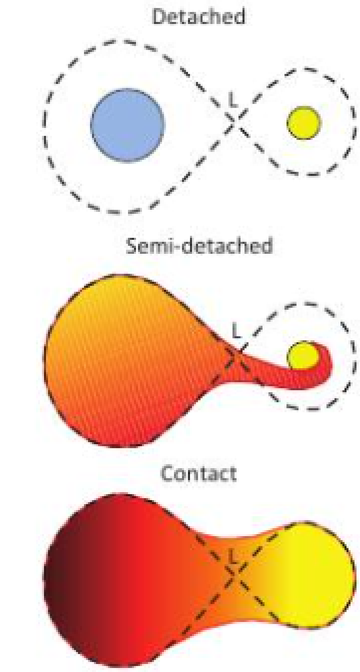
Binary stars are classified based on how we observe them:

- **Visual binaries:** These are the simplest to understand. If you see two stars orbiting each other, then it is a visual binary! However, most stars tend to be so far away, or one of the components is too faint, for the system to be resolved as two distinct stars.
- **Astrometric binaries:** These are similar to visual binaries, except only the brighter star is observable. Then, by detecting variations in its orbit, we can calculate the mass of its invisible companion.
- **Spectroscopic binaries:** These systems cannot be resolved visually. Instead, we rely on the **Doppler effect** to detect the binary system! When the brighter star moves towards us in its orbit, the light emitted from the star will be blueshifted slightly, and when the star moves away from us, the light will be redshifted slightly. By measuring these variations in the spectrum, we can calculate the period of the orbit and even the mass!
- **Photometric binaries:** We rely on changes in brightness during the orbit to detect these systems. We can classify these binaries even further depending on the reason for the change in brightness:
  - **Algol stars:** In these binaries, the dips in brightness are caused by eclipses. When the two stars are separate, both of their luminosities contribute to the total luminosity. When the brighter star (called the **primary**) eclipses the fainter companion (called the **secondary**), that creates a small dip in brightness since we no longer see light from the secondary. When the secondary eclipses the primary, we still get the brightness from the secondary, but we don't get any brightness from the portion of the primary that is covered up be the secondary. Since the primary is brighter than the secondary, this creates a larger dip in brightness. We will do an example problem in Mathz to explain how we can calculate the properties of the system from these observations.
  - β **Lyrae stars:** These systems are so close that the stars pull each other into ellipsoid shapes. As a result, the brightness varies *continuously* as the shape of the stars change during the orbit. The graph of brightness will still have two dips, but now the curve will be smoothed out.
  - **W UMa stars:** In these systems, the two dips in the light curve have nearly the same minima. These systems are usually **contact binaries**. Additional variations in the light curve can be caused by many effects (shape of stars distorted by tidal effects, limb darkening, gravity darkening, reflection of stars off each other, mass transfer).

We also classify binaries based on how the two stars share their mass:

- The **Roche lobe** is the region of space around a star where the gravity of the star causes any mass withing the Roche lobe to belong to that star. For example, if a moon falls within the Roche lobe of its planet, tidal forces from the planet will tear the moon apart until it all falls into the planet. There are 3 types of binaries based on how their Roche lobes interact:
- **Detached:** Each star lies within their Roche lobe. Therefore, no mass transfer occurs and the two stars are "detached" from one another.

- **Semi-detached:** One star exceeds its Roche lobe (like what happens when a RGB grows too big). As a result, mass transfers from that star to its smaller companion star.
- **Contact:** Both stars exceed their Roche lobe. As a result, the surfaces of the two stars contact each other and the stars act as conjoined twins.



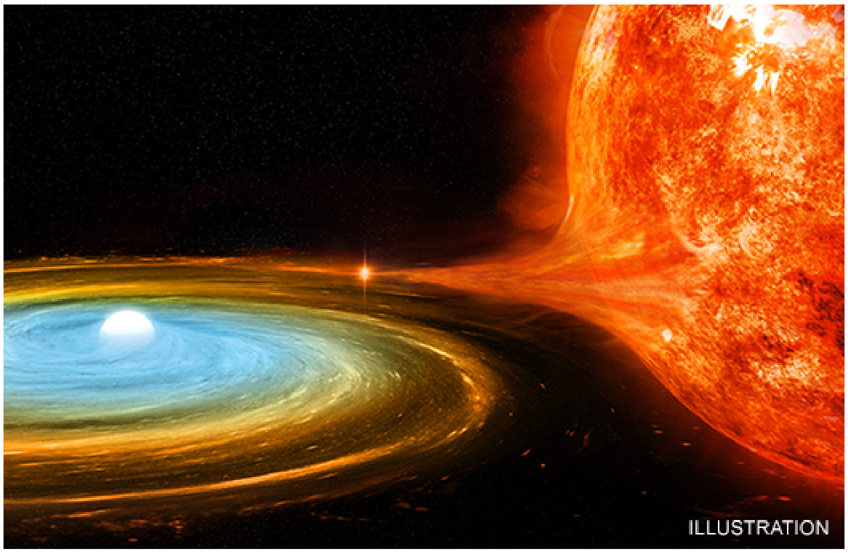


Figure 18 (left): Types of binaries
Source: Spectral Atlas for Amateur Astronomers by Richard Walker
Figure 19 (right): Illustration of a binary system where a white dwarf accretes matter from its companion star (Source: Chandra)

5.2 Binary evolution

Most of the interesting binary systems you will see involve a **white dwarf** and a companion star, usually a **red dwarf**. However, sometimes we see a white dwarf with a **blue main-sequence** star. This is super weird because we would expect the more massive star to evolve and become a white dwarf first. So how is there still a blue star?

1. Two stars are in a binary system. We'll just say one is a blue main sequence, Bob, and the other is a red main sequence, Carl.
2. The Bob evolves first, becoming a red giant. At this stage, it exceeds its Roche lobe and begins feeding matter to Carl.
3. Bob has been stripped of its outer layers, leaving behind a white dwarf. Meanwhile, Carl has just increased its mass. As a result, it is a massive, and therefore blue, main sequence star.
4. Carl will live out their life and eventually die to become a stellar remnant, likely a white dwarf. At this point, we will have two white dwarfs orbiting each other.

This general process also applies to a bunch of other scenarios (neutron star binaries, black hole binaries, etc.).

5.3 Recurrent Novae

WDs in a binary system can release explosions of energy in two ways:

1. When matter from its companion star falls onto the **accretion disk**, it heats up due to friction and radiates energy.

2. When matter from the accretion disk finally reaches the surface of the WD, it suddenly becomes hot enough to achieve fusion! This creates a flash of energy. Since it can happen multiple times, these are often known as **recurrent novae**.

5.4 Mathz

Binary systems provide a wealth of information that we would not be able to detect from a singular body. Kepler's laws can be extremely useful when finding info on the components of the binary based on their orbit:

Example 5.1: (USAAAO First Exam 2022)

5. Two (spherical) asteroids, Ek and Do, are orbiting in free space around their stationary center of mass. Ek has mass $7M_{☾}$ and Do has mass $1.4M_{☾}$, where $M_{☾}$ is the mass of moon. What is the ratio of the angular momentum of the whole system to the angular momentum of Do about the center of mass of the system?

Follow up: Now let's say that Ek and Do are stars, with masses of $7M_\odot$ and $1.4M_\odot$, respectively.

If the system is observed to have an orbital period of 8.4 yrs, what is the distance between Ek and Do? Further, what is the distance of Ek and Do, respectively, to the center of mass of the orbit?

Solution: This problem requires 1 key insight - **the angular velocity for both components of a binary is the same**. This is because the centripetal force is provided by gravity. Since gravity acts between the two bodies, they must always be on opposite sides of each other in the orbit (since centripetal force must point towards the center of orbit).

Now, let's use this idea to solve this question. Since the centripetal (gravitational) force is the same for both objects (by Newton's 3rd law), then we know:

$$m_1\omega_1^2 r_1 = m_2\omega_2^2 r$$

Since $\omega_1 = \omega_2$ (this is our insight), that means

$$m_1 r_1 = m_2 r_2$$

This means the more massive an object is, the closer it is to the center of orbit. If you are familiar with **mass points**, it's the same concept!

Plugging our values for the mass is, we see that Ek, which is 5 times as massive ($7 = 1.4 \times 5$), must have an orbit that is 5 times as close to the center of mass. Now, to solve for angular momentum we use the formula

$$L = I\omega = mr^2\omega = (mr)r\omega$$

Since we solved that mr and ω are the same for both, this means that Do will have 5 times the angular momentum of Ek. Therefore, our answer is $\frac{5+1}{5} = \frac{6}{5}$, which is **C**!

Now, for the bonus question, we need to use Kepler's third law:

$$a^3 \propto Mp^3$$

Where a is the semimajor-axis of the orbit, p is the period of the orbit, and M is the combined mass of the system. This form is very helpful since we can always compare everything to Earth's orbit, which has $a = 1\text{AU}$, $p = 1\text{yr}$, and $M = 1M_\odot$ Thus, we can just plug in the numbers the problem gave us to get

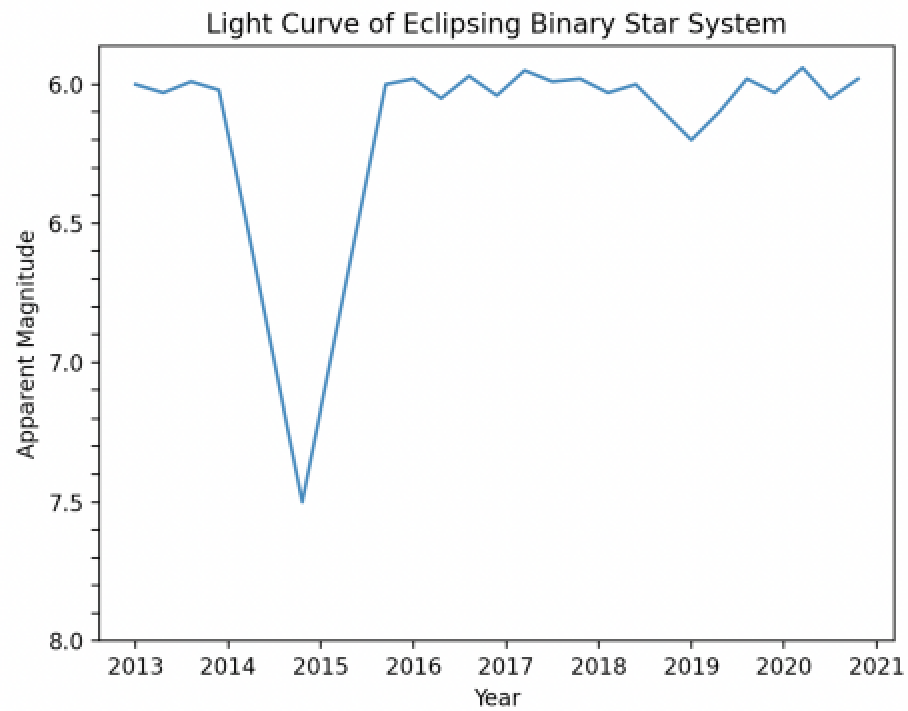
$$a^3 = 8.4(8.4)^2 = 8.4^3$$

Therefore, the distance between Ek and Do is $2a$, or 16.8AU! Since we already established that the radii of Ek's and Do's orbits are in a $1:5$ ratio, that means the distances are **2.8AU for Ek, and 14AU for Do**.

Orbit dynamics questions involving binary systems will use some variation of these questions. So, if you understand how we solved this question you should be in good shape!

Now it's time to look at a question about a binary system's *luminosity*:

Example 5.2: (USAAAO First Exam 2022)
17. An astronomer observes an eclipsing binary star system from Earth, and he plots the following light curve.



Suppose that both stars have circular orbits and the distance between the stars is 14.8 AU. What is the total mass of the binary star system in terms of solar masses?

18. Assume that the smaller star in the above binary star system is brighter than the larger star. What is the ratio of the radius of the smaller star to the radius of the larger star?

Solution: A light curve gives us 2 important pieces of info - the **brightness** and the **time**.

Since we are given the distance and we can find the period by looking at the graph, we can just use Kepler's 3rd law to find the total mass of the system! To find the period, just find the time between two dips. This will be half of the period. The primary dip happens a bit before 2015 and the secondary dip happens around 2019, which gives us a time of ~ 4.2 yrs. Thus, the total period is $4.2 \times 2 = 8.4$ yrs!

$$14.8^3 = M(8.4)^2$$

$$\text{So, } M = \frac{14.8^3}{8.4^2} \sim 46 M_{\odot} \quad \square$$

Now, for the next question, we need to make use of the luminosity shown in the graph. There are three luminosities we need to consider:

1. The combined luminosity of both stars. This would be when they appear side by side to us in their orbit. It is represented by the flat line at the top of the orbit, so ~ 6 in this case.
2. The luminosity of the larger star. This is when the larger star eclipses and completely covers the smaller star so that we no longer receive any light from it. Therefore, there will be a dip in brightness since it's only the larger star's luminosity rather than the combined luminosity. This can be either the larger (if the smaller star is brighter) or smaller dip (if the larger star is brighter) in our graph. Here, it is the larger dip (since the smaller star is brighter) and appears to be ~ 7.5
3. The luminosity of the smaller star plus *part* of the larger star. This is when the smaller star eclipses, but does *not* completely cover the larger star. Again, this dip in brightness can be either the larger (if the larger star is brighter) or smaller dip (if the smaller star is brighter) in our graph. Here, it is the smaller dip (since the smaller star is brighter) and appears to be ~ 6.2

We can use the definition of magnitudes (sorta like distance modulus) to compare the luminosities in different cases. Start of by comparing the combined luminosity to when the smaller star is eclipsed, so between 1 and 2 in this case. We see:

$$10^{-\frac{2}{5}(7.5-6)} \sim .25$$

This means that the larger star contributed $\frac{1}{4}$ of the combined luminosity, so the smaller star contributed $\frac{3}{4}$ of the combined luminosity.

Now, let's compare when the larger star is partially eclipsed.

$$10^{-\frac{2}{5}(6-6.2)} \sim .83$$

If the larger star contributes .25 of the combined luminosity, then that means $\frac{1-.83}{.25} = \frac{.17}{.25} = 68\%$ of the larger star was covered. Since $A \propto r^2$, we take the sqrt of this to find the ratio of the radii.

$$\sqrt{.68} \sim .82 \quad \square$$

6 Conclusion

Stars, like teenagers, live complicated lives. However, by understanding a few general concepts and rules, we were able to make sense of it all! This reflects a broader trend in astronomy: astronomers are able to use their ingenuity to interpret their observations. From these observations, they create simple rules that allow us to describe THE ENTIRE UNIVERSE!

Given you've made it this far, it's clear that you're interested in astro, so here's some advice: seek to understand *why* things happen; once you understand the rules that govern reality, the rest becomes intuitive. Good luck in your pursuit of knowledge!

7 Resources

For a better explanation of Hawking radiation, check out this video! ScienceClic takes confusing concepts in astronomy and quantum physics and explains our weird reality in a form that is easy to understand. If stellar remnants interest you, then I highly recommend checking out this channel: https://www.youtube.com/watch?v=isezfMo8kWQ